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ON THE ISOLATED CHARACTER OF SOLUTIONS WITH A STRONG ATTACHED SHOCK WAVE AT THE EDGES OF A V-SHAPED WING AND WEDGE*

A.V. GRISHIN

The transonic approximation is used to study the conical problems of supersonic flow past an infinite wedge and a *V*-shaped wing, the flow behind the attached shock wave is subsonic. The possibility of the existence of a flow with a strong shock in a plane perpendicular to the edge of the wing or wedge is clarified. For this reason the linear theory is used to study the boundary value problems for the perturbations in exact solutions with a plane shock. It is shown that the boundary value problems have a solution, provided that the plane shock wave corresponding to the exact solution is weak (in a plane perpendicular to the edge), and have no solution when the shock is strong.

Earlier /1/, the problem of flow past a *V*-shaped wing was studied, where the flow was supersonic behind the attached shock wave. Experimental investigations /2—4/ of the flow past a *V*-shaped wing resembling flows with a strong plane shock, made it possible to establish /4/ the isolated nature of the flow with a strong shock. Numerical methods /5/ and experimental methods /2—4/ were used to show that when the angle of attach of the *V*-shaped wing with a strong plane shock is reduced, a flow results with Mach interaction between the shock waves and the weak discontinuity at the edge. It was established /4/ that increasing the angle of attack causes detachment of the shock wave. A non-steady model of supersonic flow past an infinite wedge is proposed in /6/** (**see also Rusanov V.V. and Sharakshane A.A. Non-steady models of flow past conical bodies. Preprint In-ta prikl. matem. Akad. Nauk SSSR, Moscow, No.27, 1978, and Rusanov V.V. and Sharakshane A.A., Study of a linearized non-stationary model of flow past an infinite wedge. Preprint In-ta prikl. matem. Akad. Nauk SSSR, Moscow, No.103, 1980) and it is shown that a flow with a strong shock wave is unstable in this model. The non-existence of a flow with a strong shock represents a basically different case of a finite wedge and was proved using the hodograph method /7/ without taking into account the vorticity (in the transonic approximation). The result is generalized in /8/ to the case of vortical flows.

1. Assuming that the velocities are normalized with respect to the speed of sound, we shall consider the problems in the transonic approximation. We take, as the unperturbed flow, the uniform flow past a wedge with a strong or weak shock wave attached to the edge of the wedge, in the case when the flow behind the shock is subsonic. We choose a coordinate system attached to the edge of the wedge, in which the z axis is directed along the edge and the x axis along the velocity vector behind the shock wave (Fig.1). The transonic velocity components $\mathbf{v} = (1 + u, v, w)$ can then be represented in the form $u = u_{01}, v = v_{01}, w = 0$ in front of the wave,

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and $u = u_{02}$, $v = 0$, $w = 0$ behind the wave.

The flow with a plane attached shock wave at the wedge yields exact solutions for a family of V-shaped wings with a plane attached shock. Indeed, let us draw straight lines along the shock wave from the origin of coordinate. The stream lines emerging from the points lying on these straight lines form a V-shaped surface whose edge lies on the wedge and coincides with the x axis. The surface can be regarded as a V-shaped wing with a plane shock at the edges.

Let us assume that a perturbation has been introduced into the concoming flow by slightly varying the velocity component u, and that the surface of the attached shock wave has become slightly curved. Then the velocity components in front of the wave will be $u = (1+\varepsilon)u_{01}$, $v = v_{01}$, $w = 0$ where ε is a small parameter, and behind the wave they will be $u = u_{02} + u'$, $v = v'$, $w = w'$. We shall also assume that this flow, as well as the unperturbed flow, will have conical symmetry and all flow parameters will depend on the variables $\xi = z/x$, $\eta = y/x$. We shall write the equation of the shock wave in the form $\eta = \eta_0 + \eta'$ where η_0 corresponds to the initial position of the shock wave. We shall choose η_0 and u_{02} as the independent parameters, and confine ourselves to considering wings symmetrical with respect to the xy-plane. The geometry of the wing will be specified by the parameters η_0 and β (Fig.1), and we will write $u_{02} = -k^2$, $k\eta_0 = c$.

We follow /1/ is formulating the boundary value problem for the region OAB in the $\xi\eta$ plane, where OA represents the wing surface, AB the shock wave and BO the planes of symmetry. Since the problem is studied in the transonic approximation, a conical potential exists. The potential equation yields, after linearization, an equation for w' /1/ in (/1/ $u_{02} = k^2$)

$$(1-k^2\xi^2)w'_{\xi\xi} + 2k^2\xi\eta w'_{\xi\eta} + (1-k^2\eta^2)w'_{\eta\eta} = -2k^2(\xi w_\xi' + \eta w_\eta') \qquad (1.1)$$

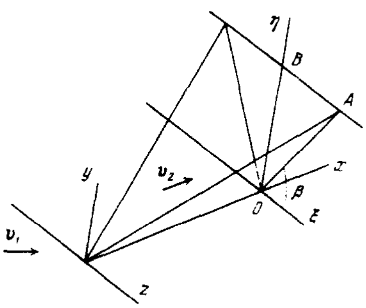

Fig.1

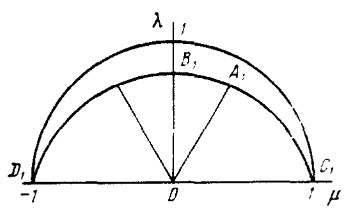

Fig.2

The equation of the shock polar and condition of potential continuity after its linearization, split into the corresponding conditions for the parameters of the unperturbed flow, and for the perturbations, and these in turn yield the condition for w' at the shock wave /1/

$$\eta = \eta_0,\quad \frac{\xi}{\eta_0}\frac{1-c^2}{1+c^2}\, w_\xi' - w_\eta' = 0 \qquad (1.2)$$

We shall also require the condition of continuity of the perturbation potential /1/

$$\eta = \eta_0,\quad \varepsilon u_{01} + \eta' v_{01} = u' + \eta_0 v' + \xi w' \qquad (1.3)$$

We specify at the wing surface the condition of impermeability $v' = w'\,\mathrm{tg}\,\beta$ which yields, after differentiating along the wing surface,

$$\eta - \xi\,\mathrm{tg}\,\beta = 0,\quad \left(2 - \frac{k^2\xi^2}{\cos^2\beta}\right)\mathrm{tg}\,\beta\, w_\xi' - \left(1 - \mathrm{tg}^2\beta - \frac{k^2\xi^2\,\mathrm{tg}^2\beta}{\cos^2\beta}\right) w_\eta' = 0 \qquad (1.4)$$

Therefore we have, at the plane of symmetry $w' = 0$,

$$\xi = 0,\quad w_\eta' = 0 \qquad (1.5)$$

The coordinate transformation

$$\xi = \frac{2}{k}\frac{\mu}{1-\mu^2-\lambda^2},\quad \eta = \frac{2}{k}\frac{\lambda}{1-\mu^2-\lambda^2} \qquad (1.6)$$

reduces (1.1) to the Laplace equation. The straight line $\eta = \eta_0$ becomes a circular arc $k\eta_0(1-\mu^2-\lambda^2) = 2\lambda$, region OAB maps onto the region OA_1B_1 and the upper half-plane maps into half of the unit circle (Fig.2). The boundary conditions (1.2), (1.4), (1.5) become, respectively,

$$(1 - c^2 + 2c\lambda)\, w_\mu' - [2c\mu + (1+c^2)\lambda\mu^{-1}]\, w_\lambda' = 0 \qquad (1.7)$$

$$2\,\mathrm{tg}\,\beta\cos^4\beta\, w_\mu' - [(1-\mathrm{tg}^2\beta)\cos^4\beta - \mu^2]\, w_\lambda' = 0 \qquad (1.8)$$

$$w_\lambda' = 0. \qquad (1.9)$$

The boundary value problem formulated in the region OA_1B_1 represents a homogeneous Hilbert problem for the analytic function $f + ig = w_\mu - iw_\lambda$, with a boundary condition of the form $Sf + Lg = 0$ discontinuous at the apices of the region. The solvability of the problem is established using the conformal transformation $r = R(\omega) = R(\mu + i\lambda)$ to map the region into a circle and calculating the index of the corresponding problem for the function $f_1(r) + ig_1(r) = f(\omega) + ig(\omega)$ with boundary condition $S_1(t)f_1(t) + L_1(t)g_1(t) = 0$, $S_1(t) = S(\tau)$, $L_1(t) = L(\tau)$ at the circumference.

The index $\varkappa$ of the problem is given by the formula $\varkappa = \sum\varkappa_k$ where $\varkappa_k$ are integers calculated at the points where the boundary condition is discontinuous, depending on the class of functions required at these points (we have in mind the index of the equivalent Riemann problem with a continuous coefficient, whose solvability is established using well-known theorems /9/).

The class of functions $f+ig$ is specified by the physical condition that the velocity w' is bounded, which is equivalent to the integrability of $f+ig$ in the $\mu\lambda$ plane. To choose the class of f_1+ig_1 at the point on the curcumference at which the argument of the vector (S_1, L_1) is discontinuous, we must take into account the asymptotic forms of the mapping $r=R(\omega)$ defined by the internal angles α_k at the apices of the region OA_1B_1. To have w' bounded, we must choose the numbers $\varkappa_k$ using the conditions /1/

$$\varkappa_k = \begin{cases} 1, & \pi-\alpha_k<\theta_k<\pi \\ 0, & -\alpha_k<\theta_k<\pi-\alpha_k \\ -1, & -\pi<\theta_k<-\alpha_k \end{cases}$$

where θ_k is the jump in the argument of the vector (S_1, L_1) at the discontinuity in question (the argument is calculated as $\operatorname{arctg}(L_1/S_1)$, with $|\theta_k|<\pi$). The branches of the argument chosen in this manner have, by virtue of (1.7)-(1.9), discontinuities only at the points O and A_1.

Calculating now the index with respect to the values of the parameters η_0, u_{02}, β we find, that when $2\sin^2\beta>1-c^2$, we have $\varkappa=-1$ and no non-trivial solutions of the problem exist /9/. When $2\sin^2\beta<1-c^2$, we have $\varkappa=0$. i.e. in accordance with /9/ the problem has a unique solution, defined apart from a constant multiplier. The multiplier is therefore obtained from the condition that the shock wave is attached ($\eta'=0$ at the point A) and condition $w'=0$ in the plane symmetry.

From the equation of the shock polar in the plane perpendicular to the wing edge /1/ we find, that when $2\sin^2\beta>1-c^2$, then the shock wave is strong, and weak when $2\sin^2\beta<1-c^2$. Consequently, if the shock wave is strong, the problem has no solution, while when the shock wave is weak, a solution exists and is unique.

2. Let us consider the problem of the flow past a wedge. We introduce a perturbation into the initial flow with a plane shock at the wedge, by means of a slight conical deformation of the wedge surface ($\varepsilon=0$). Then we have the condition of impermeability at the surface $v=q(\xi)$. We shall consider the perturbations under which $dq/d\xi$ satisfies the Hölder condition. Let us assume for simplicity that $q(\xi)=0$ when $|\xi|>\xi_0>0$. We shall determine whether a solution exists for an arbitrary function $q(\xi)$ belonging to the class in question. We shall seek a solution under the condition that the magnitude of the perturbation of the shock wave η' is finite over its whole extent.

Let us formulate the boundary value problem in the strip $0<\eta<\eta_0$ bounded by the images of the wedge surface $\eta=0$ and of the shock wave $\eta=\eta_0$. Differentiating the condition of impermeability along the wedge surface, we find

$$\eta=0,\ w_\eta'=v_\xi'=dq/d\xi=\rho(\xi).$$

Condition (1.2) holds at the surface of the shock wave.

Changing now to the variables μ, λ, we obtain an inhomogeneous boundary value Hilbert problem for the function $f+ig=w_\mu'-iw_\lambda'$ in the region $OC_1B_1D_1$, with the condition

$$\lambda=0,\ w_\lambda'=\rho(\xi(\mu))=p(\mu)$$

on D_1C_1, and condition (1.8) on $C_1B_1D_1$. The conditions have the form $Sf-Lg=p$, and the argument of the vector (S, L) has discontinuities at the points C_1D_1.

To find the index of the equivalent Riemann problem, we must find out in which class of functions its solution should be sought. The condition of continuity of the potential (1.3) shows that for η' to be bounded as $|\xi|\to\infty$, the quantity w' must have an asymptotic of the form $w'\sim 1/\xi^{1+\alpha}$ where $\alpha>0$. Taking into account the asymptotic form of the mapping (1.6) at the points C_1, D_1 we find that in order to satisfy this requirement the function $f-ig$ must have zeros of a certain order at these points. Since the argument of the vector (S, L) has first-order discontinuities at the points C_1 and D_1, the presence of zeros will be ensured if the solution is sought in the class of bounded functions /9/.

Mapping now the region $OC_1B_1D_1$ into a circle and calculating the index of the corresponding Riemann problem in the class of bounded functions (the mapping changes only the order of the zeros of the solution at the images of the points C_1, D_1). we find that $\varkappa=-2$ when $1-c^2<0$. i.e. the problem has no solution when the function $q(\xi)$ is arbitrary. A solution exists and is unique, if $q(\xi)$ satisfies one condition of solvability /9/. This condition is satisfied automatically, provided that $q(\xi)=q(-\xi)$. When $1-c^2>0$, the index is equal to zero and the problem has a general solution of the form $f+ig=F(\omega)+CQ(\omega)$, in the $\mu\lambda$ plane where C is an arbitrary real constant /9/. To satisfy the condition $w=0$ at the points C_1, D_1, the solution obtained must satisfy the additional requirement that

$$\operatorname{Re}\int_{C_1}^{D_1}(f-ig)\,d\omega=0$$

which yields the constant C. When $\varkappa = -2$, the requirement represents the second condition of solvability. The solutions obtained when both conditions of solvability are satisfied, are isolated.

Since the condition $1 - c^2 > 0$ corresponds to a weak, and $1 - c^2 < 0$ to the strong shock wave /1/, it follows from the above analysis that when the wedge surface is subjected to an arbitrary conical perturbation, a solution exists and is unique if the unperturbed shock wave is weak, and there are no solutions if the shock is strong.

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TWO-VELOCITY MECHANICS OF GRANULAR POROUS MEDIA*

N.D. MUSAYEV

A two-phase mixture of a porous or granular solid phase and liquid or gas filling the pores or the intergranular spaces, is considered. Two limiting structures of the mixture are specified: 1) the solid phase represents dense packing of spherical particles (grains) in intergranular point contact; 2) the pores represent channels, almost cylindrical in form. Expressions for the interphase forces and equations of the two-velocity motion of the phases are studied within these two structures. Different development of the interphase forces depending on the structure of the mixture is noted, the forces arising from the forces of inertia and in particular from the Archimedes and the attached-mass forces.

Using the representations of the multivelocity continuum, we shall write the equation of conservation of phase masses in the form /1/

$$\partial\rho_1/\partial t + \nabla^k\rho_1 v_1^k = J_{21},\ \ \partial\rho_2/\partial t + \nabla^k\rho_2 v_2^k = J_{12} \tag{1}$$
$$(\rho_i = \rho_i^\circ\alpha_i,\ i = 1,\ 2;\ \alpha_1 + \alpha_2 = 1)$$

The lower indices $i = 1, 2$ refer, respectively, to the parameters of the liquid (gaseous) and solid phases, ρ_i^0 and ρ_i are the real and apparent density connected with each other through the volume concentration, α_i, v_i is the velocity of the i-th phases, and J_{ji} is the phase transition intensity characterizing the amount of mass of the j-th phase transported to the i-th phase per unit volume of the mixture, in unit time $(i, j = 1, 2;\ i \neq j)$.

The equations of the phase moments can be written in the form /1/

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